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Design of Algebraic Observers for Brass Instruments

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Context

Realistic Musical Restitution
for Brass Instruments

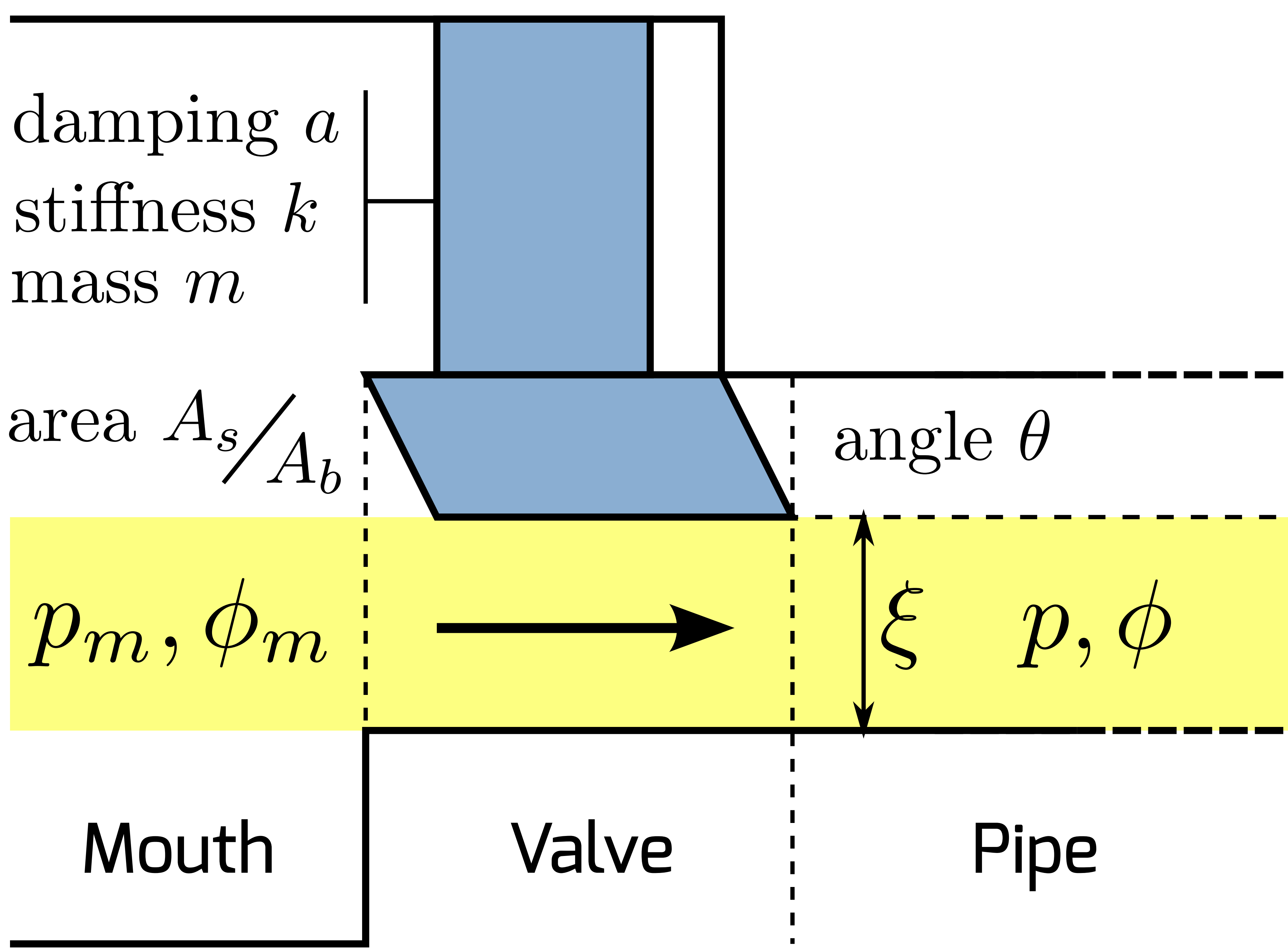
Physical Modelling
&
Estimation Problem:

State + Control + Parameters

Lip Model

$$\ddot{\xi} + 2\zeta\omega\dot{\xi} + \omega^2(\xi - \xi_e) = \omega^2 f$$

$$f = \gamma_s(p_m - p) + \gamma_j p$$



$$\omega = \sqrt{k/m}$$

$$\zeta = a/(2\sqrt{km})$$

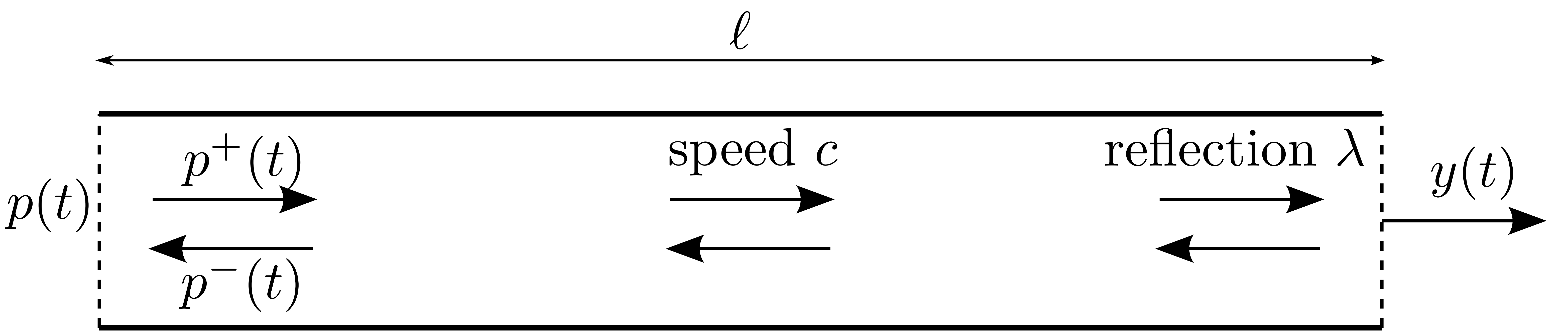
$$\gamma_s = (A_s \sin\theta)/k$$

$$\gamma_j = A_b/k$$

Acoustic Pipe

Bernoulli Equation:

$$\frac{\mu}{2}\left(\frac{p^+(t)-p^-(t)}{\xi(t)}\right)^2 = p_m(t) - p^+(t) - p^-(t)$$



$$\tau = \frac{2\ell}{c}$$

$$p^-(t) = \lambda p^+(t-\tau)$$

$$y(t) = (1+\lambda)p^+(t-\tau/2)$$

Infinite-Dim. System

$$p^+(t) = P^+(p^+(t-\tau), \xi(t), p_m(t))$$

$$\ddot{\xi}(t) = L(p^+(t-\tau), \xi(t), \dot{\xi}(t), p_m(t))$$

Functional Differential Equation with state

$(p_t^+, \xi(t), \dot{\xi}(t))$ where $p_t^+ : \theta \in [-\tau, 0] \mapsto p^+(t+\theta)$.

studied as a **Neutral Delay-Differential Equation**:

"**Asymptotic State Observers for a Simplified Brass Instrument Model**", B. d'Andréa-Novel, J.-M. Coron & T. Hélie in *Acta Acustica* 96-4 (2010).

or as a **Delay-Differential Algebraic Equation**.

Lip Height "Measure"

From the sound pressure y:

$$\xi(t)^2 = \Xi^2(p_m(t), y(t-\tau/2), y(t+\tau/2))$$

$$\Xi^2(p_m, y_+, y_-) \propto \frac{(y_+ - \lambda y_-)^2}{(p_m - y_+) + \lambda(p_m - y_-)}$$

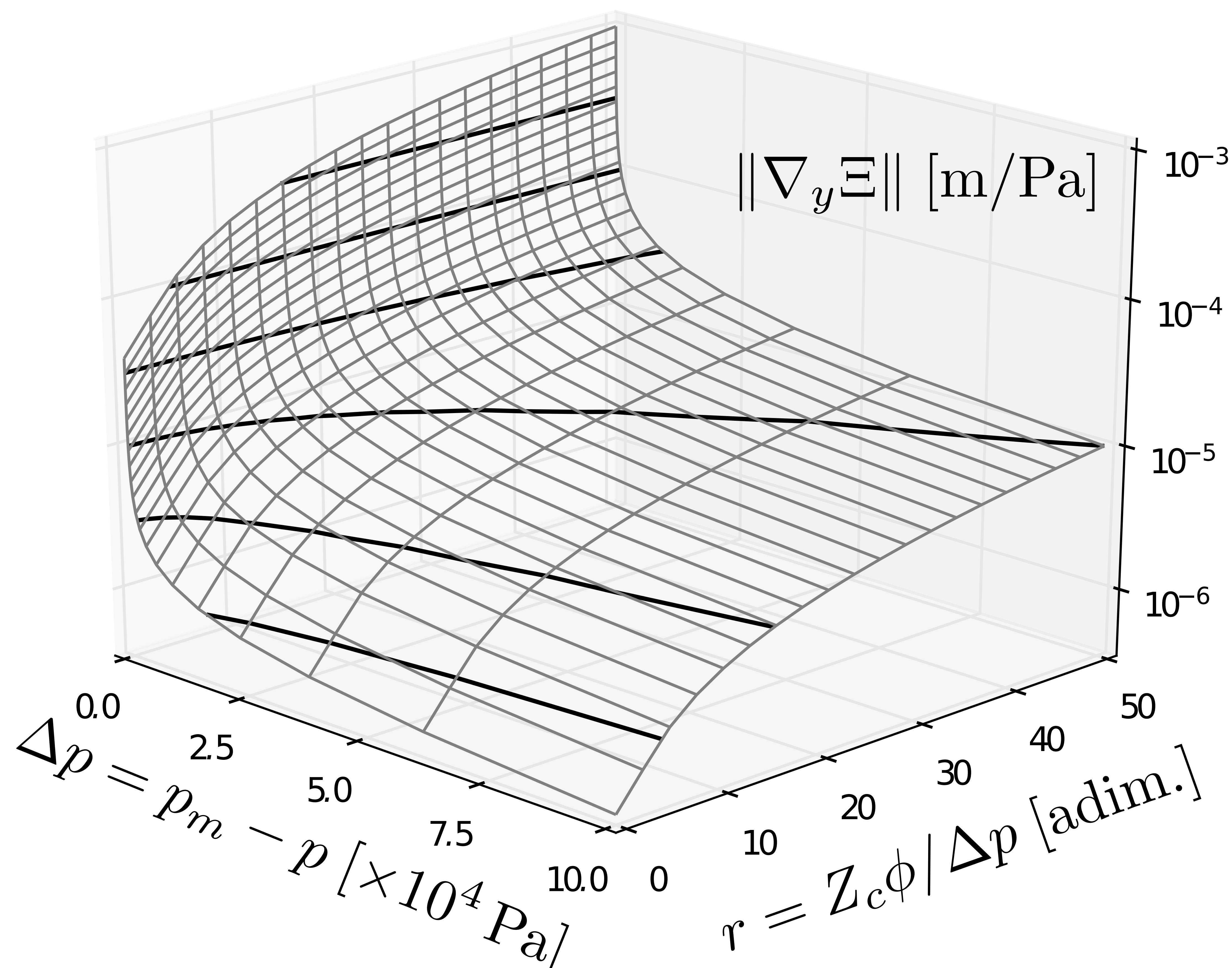
Estimation $\widehat{\xi}$ from the sound measure $\widehat{y}$.

Error Estimate

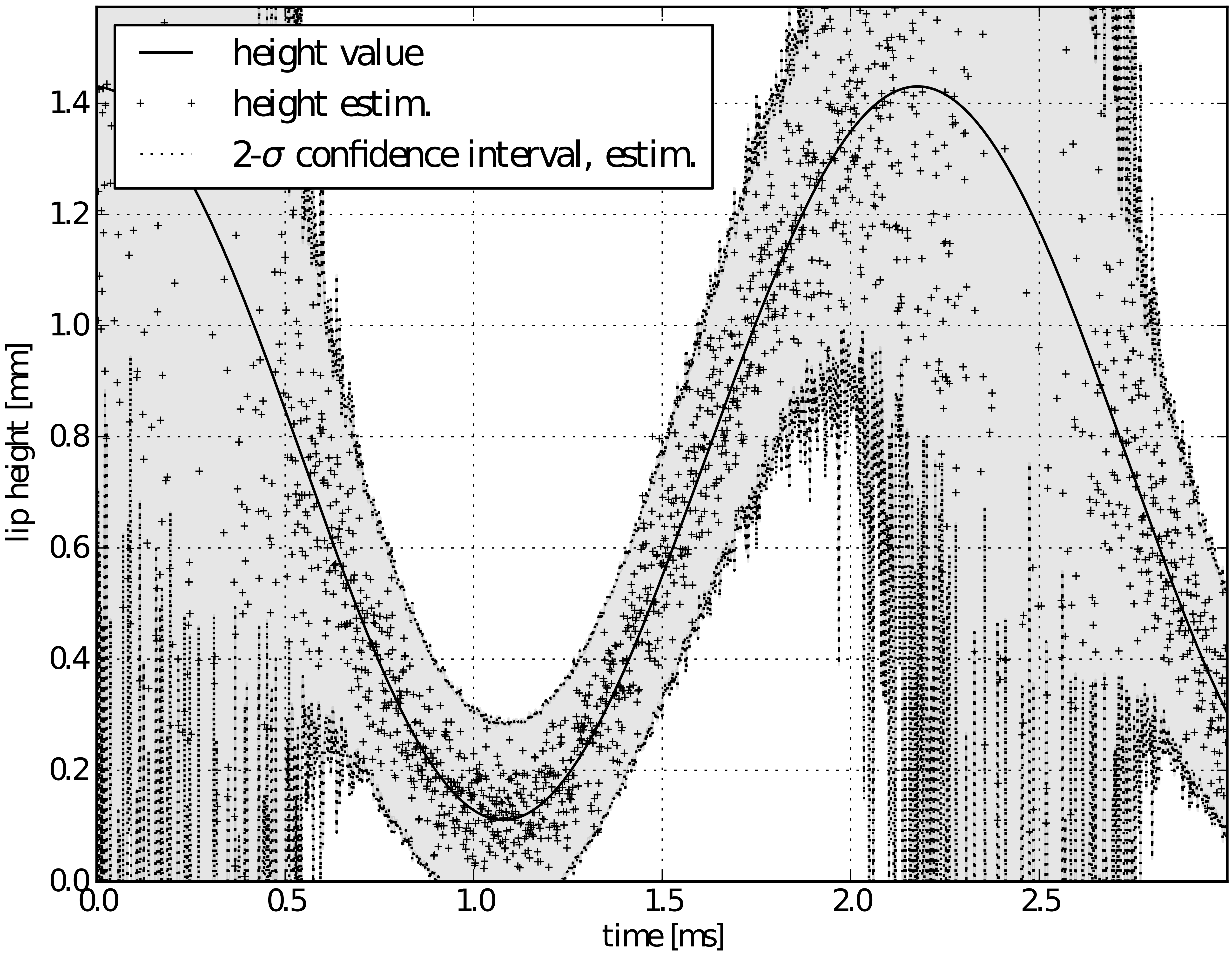
$$\widehat{\xi}(t) - \xi(t) \simeq \nabla_y \Xi(t) \cdot \left[\begin{array}{c} \widehat{y}(t-\tau/2) - y(t-\tau/2) \\ \widehat{y}(t+\tau/2) - y(t+\tau/2) \end{array} \right]$$

Sensitivity Analysis

$$\nabla_y \Xi = \sqrt{\frac{\mu}{2\Delta p}} \frac{1}{1+\lambda} \begin{bmatrix} \lambda(r/2-1) \\ (r/2+1) \end{bmatrix}$$



Height Measure Precision



Lip Dynamics

$$X(t) = \left[\begin{array}{c} \xi(t-\tau/2) \\ \dot{\xi}(t-\tau/2) \end{array} \right]$$

$$X(t+dt) = AX(t) + B\widehat{p}(t-\tau/2) + u(t) + w(t)$$

$$\widehat{\xi}(t-\tau/2) = CX(t) + v(t)$$

Sound measure perturbation:

Gaussian White Noise with constant power Σ_y.

State & output perturbation $(w(t), v(t))$**:**

$$\text{G.W.N. with} \left| \begin{array}{l} \mathbf{var}(w) = K \times \Sigma_y \\ \mathbf{var}(v)(t) = \|\nabla_y \Xi(t-\tau/2)\|^2 \times \Sigma_y \\ \mathbf{cov}(w,v)(t) = K' \nabla_y \Xi(t-\tau/2) \times \Sigma_y \end{array} \right.$$

Observer Design

Kalman Filter Equations:

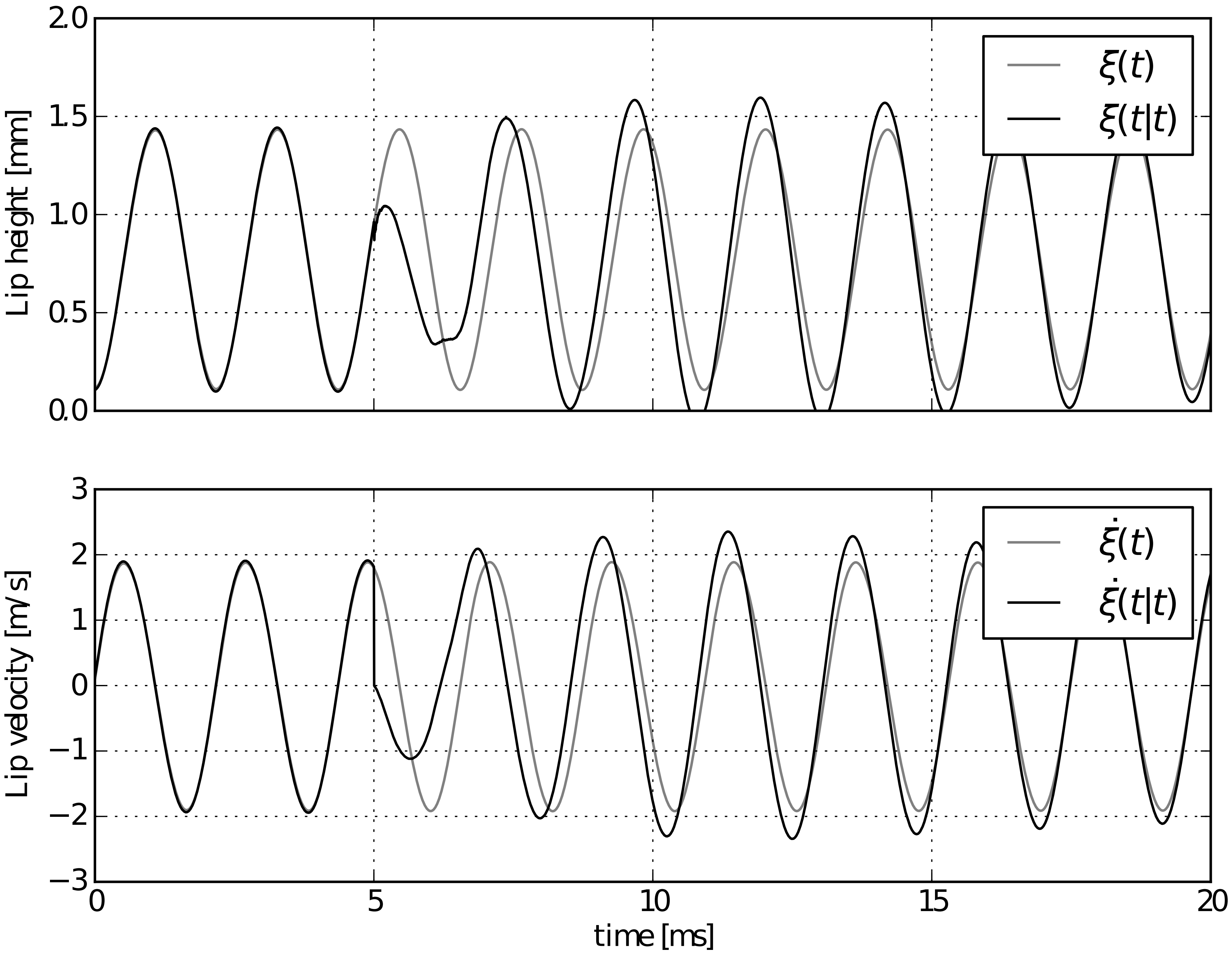
- State estimate $X(t|t)$ of $X(t)$.
- Error estimate $\mathbf{var}[X(t) - X(t|t)]$.

"**Analysis of Kalman filter with correlated noises under different dependence**", L. Ma, H. Wang & J. Chen in J. Inf. Comput. Sci. 7.5, pp. 1147-1154 (2010).

Concrete Design:

- The noise covariance is estimated,
- "Out-of-range" measures are managed.

Experimental Validation



Context & Results

- SNR = 12 dB
- steady-state error < 1 %
- fast transitional behavior after observer reset.

Conclusion

Algebraic observer scheme:

- simple configuration process,
- robust w.r.t. large noise power.

Future

- hybrid (open/closed) lip model,
- model parameters estimation.